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MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2018

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the ratio in which the point C, (9, 8, -10) divides the line segment joining the points A(5, 4, -6) and B(3, 2, -4). **(06 Marks)**

b. If cos α, cos β, cos γ are the direction cosines of a straight line, prove that (i) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ (ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$. **(07 Marks)**

c. Find the constant K such that the angle between the lines with direction ratios (-2, 1, -1) and (1, -K, 1) is 90°. **(07 Marks)**

2 a. Show that the angles between the diagonals of a cube is $\theta = \cos^{-1}(1/3)$. **(06 Marks)**

b. Find the equation of the plane through the points (1, 0, -1) and (3, 2, 2) and parallel to the line $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-2}{3}$. **(07 Marks)**

c. Show that the points A(-6, 3, 2), B(3, -2, 4), C(5, 7, 3) and D(-13, 17, -1) are coplanar. Also find the equation of the plane containing them. **(07 Marks)**

3 a. Find the angle between the vectors $\vec{a} = 2i + 6j + 3k$, $\vec{b} = 12i - 4j + 3k$. **(06 Marks)**

b. Find the area of a parallelogram whose adjacent sides are $i - 2j + 3k$ and $2i + j - 4k$. **(07 Marks)**

c. Find a unit vector perpendicular to both vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = 7i - 5j + k$. **(07 Marks)**

4 a. Show that the four points whose position vectors are $3i - 2j + 4k$, $6i + 3j + k$, $5i + 7j + 3k$ and $2i + 2j + 6k$ are coplanar. **(06 Marks)**

b. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$ where t is the time. Find the components of velocity and acceleration at t = 1 in the direction of i + j + 3k. **(07 Marks)**

c. Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of vector i + 2j + 2k. **(07 Marks)**

5 a. Find div F and curl F where $F = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. **(06 Marks)**

b. Show that $F = x(y - z)i + y(z - x)j + z(x - y)k$ is solenoidal. **(07 Marks)**

c. Find the constants a and b so that the vector $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational. **(07 Marks)**

6 a. Find the Laplace transforms of $1 + 2t^3 - 4e^{3t} + 5e^{-t}$. **(07 Marks)**

b. Find the Laplace transform of $t^2 \sin^2 t$. **(07 Marks)**

c. Find the Laplace transform of $\frac{\sin at}{t}$. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

7 a. Find the inverse Laplace transform of $\frac{3s-4}{16-s^2}$. (06 Marks)

b. Find the inverse Laplace transform of $\frac{1}{s^2+4s+9}$. (07 Marks)

c. Evaluate $L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\}$. (07 Marks)

8 a. Obtain the Laplace transforms of $f'(t)$, $f''(t)$. (08 Marks)

b. Solve the differential equation using Laplace transforms $y''-3y'+2y=1-e^{2t}$ under the conditions $y(0)=1$, $y'(0)=0$. (12 Marks)

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